Mean-field modeling of crowd dynamics

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- The topic of my PhD project is mean-field type control and games in crowd dynamics.
- This presentation is focused on congestion aversion in pedestrian crowds.

Götgatan, Stockholm

KTH

Outline

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Pedestrian crowd modeling

Typical pedestrian behavior [Cristiani, Piccoli, and Tosin 2010]:

- Will to reach specific targets.
- Repulsion from other individuals.
- Deterministic if the crowd is sparse, partially random if the crowd is dense.

Classical models for interacting particle systems yield this behavior, but the dynamics are ruled by inertia, i.e. *a priori fixed*. Real pedestrians are "smart" and follow decision-based dynamics.

Pedestrian crowd modeling

- Particle system
  - Robust - interaction only through collisions
  - Blindness - dynamics ruled by inertia
  - Local - interaction is pointwise
  - Isotropy - all directions equally influential
- "Smart" agent
  - Fragile - avoidance of collisions and obstacles
  - Vision - dynamics ruled at least partially by decision
  - Nonlocal - interaction at a distance
  - Anisotropy - some directions more influential than others

Comparison by [Cristiani, Piccoli, and Tosin 2010].

Pedestrian crowd modeling

Perspectives on pedestrian crowd dynamics:

- Microscopic
  - the social force model [Helbing and Molnar 1995].
- Macroscopic
  - fluid-dynamic model [Hughes 2003].
- Multiscale,
  - embedding of micro and macro scales [Cristiani, Piccoli, and Tosin 2010].

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Many possible extensions:
controlled noise, multiple interacting crowds, fast exit times, interaction with environment.

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- The mean-field heuristic: as $N \to \infty$ the empirical measure converges to a probability law with which all pedestrians interact.

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Research question

The mean-field approach introduced in [Lachapelle and Wolfram 2011]:

$$\begin{cases} \min_a \displaystyle\int_{\mathbb{R}^d}\int_0^T \frac{1}{2}|a(t,x)|^2 m(t,x) + m^2(t,x)dt + \Psi(x)m(T,x)dx \\ \text{s.t. } \dfrac{\partial m}{\partial t} = \dfrac{\sigma^2}{2}\Delta m - \nabla\cdot(am),\ m(0,x) = m_0(x), \end{cases}$$

where m_0 is a probability density function.

We wanted to investigate...

- What is the probabilistic interpretation of the model?
- Especially, what is the interpretation of the term m^2?

The interpretation: local congestion penalty

Consider a crowd of N pedestrians,

$$\begin{cases} \min_{a^i} \ \mathbb{E}\left[\int_0^T \frac{1}{2}|a^i(t,X_t^i)|^2 + \int_{\mathbb{R}^d} \phi_r(X_t^i - y)\mu_t^N(dy)dt + \Psi(X_T^i)\right], \\ \text{s.t. } \ dX_t^i = a_t^i(t,X_t^i)dt + \sigma dW_t^i, \ X_0^i = \xi^i, \end{cases}$$

where ϕ_r is a (smoothed and normalized) indicator function on $B_r(0)$.

- Under an anonymity assumption, taking the limit $N \to \infty$ gives a mean-field approximation.
- Then taking the limit $r \to 0$ we retrieve the model of [Lachapelle and Wolfram 2011].

Results

In [Aurell and Djehiche 2017 (preprint)] we extend the model of [Lachapelle and Wolfram 2011].

Under the assumption of anonymous pedestrians, it contains

- A probabilistic interpretation of non-local congestion
- Pontryagin type maximum principle that characterizes the optimal control for the mean-field approximation of
    - a crowd controlled by a central planner
    - a game between arbitrarily (but finitely) such crowds

Outline

Local vs. non-local congestion avoidance

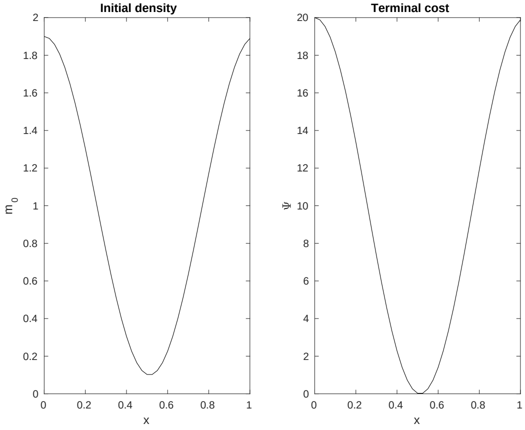
Consider the following two crowd models:

$$\begin{cases} \min_a & \int_{\mathbb{T}} \int_0^T \left(\frac{a^2(t,x)}{2} + C \int_{\mathbb{T}} \phi_r(x-y) m(t,y) dy \right) m(t,x) dt + \Psi(x) m(T,x) dx, \\ \text{s.t.} & \dot{m}(t,x) = \frac{1}{2} m''(t,x) - (a(t,x)m(t,x))', \\ & m(0,x) = m_0(x). \end{cases} \quad \text{(Non-local)}$$

$$\begin{cases} \min_a & \int_{\mathbb{T}} \int_0^T \left(\frac{a^2(t,x)}{2} + Cm(t,x) \right) m(t,x) dt + \Psi(x) m(T,x) dx, \\ \text{s.t.} & \dot{m}(t,x) = \frac{1}{2} m''(t,x) - (a(t,x)m(t,x))', \\ & m(0,x) = m_0(x). \end{cases} \quad \text{(Local)}$$

For each of them, the maximum principle gives a system of PDEs that characterize the optimal feedback control.

Local vs. non-local congestion avoidance



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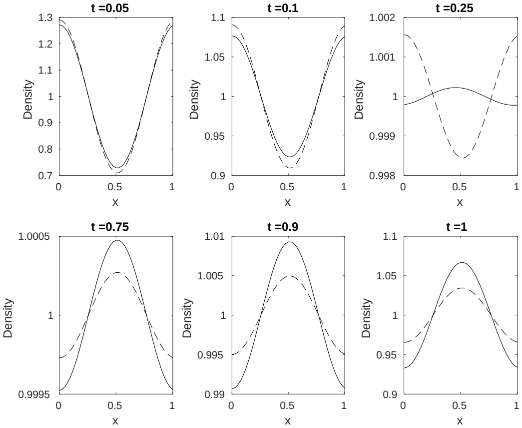

The optimally controlled density.
Dashed - local, Line - non-local.

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Thank you!